Problems

Ted Eisenberg, Section Editor

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This section of the Journal offers readers an opportunity to exchange interesting mathematical problems and solutions. Please send them to Ted Eisenberg, Department of Mathematics, Ben-Gurion University, Beer-Sheva, Israel or fax to: 972-86-477-648. Questions concerning proposals and/or solutions can be sent e-mail to <eisenbt@013.net>. Solutions to previously stated problems can be seen at <http://ssmj.tamu.edu>.

Solutions to the problems stated in this issue should be posted before February 15, 2011

- **5134:** *Proposed by Kenneth Korbin, New York, NY*

  Given isosceles $\triangle ABC$ with cevian CD such that $\triangle CDA$ and $\triangle CDB$ are also isosceles, find the value of
  $$\frac{AB}{CD} - \frac{CD}{AB}.$$

- **5135:** *Proposed by Kenneth Korbin, New York, NY*

  Find a, b, and c such that
  $$\begin{cases} ab + bc + ca = -3 \\ a^2b^2 + b^2c^2 + c^2a^2 = 9 \\ a^3b^3 + b^3c^3 + c^3a^3 = -24 \end{cases}$$
  with $a < b < c$.

- **5136:** *Proposed by Daniel Lopez Aguayo (student, Institute of Mathematics, UNAM), Morelia, Mexico*

  Prove that for every positive integer n, the real number
  $$\left(\sqrt{19} - 3\sqrt{2}\right)^{1/n} + \left(\sqrt{19} + 3\sqrt{2}\right)^{1/n}$$
  is irrational.

- **5137:** *Proposed by José Luis Díaz-Barrero, Barcelona, Spain*

  Let a, b, c be positive numbers such that $abc \ge 1$. Prove that
  $$\prod_{cyclic} \frac{1}{a^5 + b^5 + c^2} \le \frac{1}{27}.$$

- **5138:** *Proposed by José Luis Díaz-Barrero, Barcelona, Spain*

  Let $n \ge 2$ be a positive integer. Prove that
  $$\frac{n}{F_nF_{n+1}} \le \frac{1}{(n-1)F_1^2 + F_2^2} + \cdots + \frac{1}{(n-1)F_n^2 + F_1^2} \le \frac{1}{n}\sum_{k=1}^{n}\frac{1}{F_k^2},$$
  where F_n is the n^{th} Fibonacci number defined by $F_0 = 0, F_1 = 1$ and for all $n \ge 2,\ F_n = F_{n-1} + F_{n-2}$.

- **5139:** *Proposed by Ovidiu Furdui, Cluj, Romania*

  Calculate
$$\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{\zeta(n+m)-1}{n+m},$$
  where ζ denotes the Riemann Zeta function.

Solutions

- **5116:** *Proposed by Kenneth Korbin, New York, NY*

  Given square $ABCD$ with point P on side AB, and with point Q on side BC such that
$$\frac{AP}{PB}=\frac{BQ}{QC}>5.$$
  The cevians DP and DQ divide diagonal AC into three segments with each having integer length. Find those three lengths, if $AC=84$.

  Solution by David E. Manes, Oneonta, NY

  Let E and F be the points of intersection of AC with DP and DQ respectively. Then $AE=40,\ EF=37$ and $FC=7$.

  Since $ABCD$ is a square with diagonal of length 84, it follows that the sides of the square have length $42\sqrt{2}$. Let $\dfrac{AP}{PB}=\dfrac{BQ}{QC}=t>5$. Then $AP=t\cdot PB$ and $AP+PB=AB=42\sqrt{2}$. Therefore,
$$\begin{aligned} PB(t+1) &= 42\sqrt{2}\\ PB &= \frac{42\sqrt{2}}{1+t},\quad\text{and}\\ AP &= \frac{42\sqrt{2}\cdot t}{1+t}. \end{aligned}$$
  Similarly, $QC=\dfrac{42\sqrt{2}}{1+t}$ and $BQ=\dfrac{42\sqrt{2}\cdot t}{1+t}$.

  Coordinatize the problem so that
$$\begin{aligned} A &= (0,0),\quad B=(42\sqrt{2},0),\quad C=(42\sqrt{2},42\sqrt{2}),\quad D=(0,42\sqrt{2}),\\ P &= \left(\frac{42\sqrt{2}\cdot t}{1+t},0\right),\ \text{and}\quad Q=\left(42\sqrt{2},\frac{42\sqrt{2}\cdot t}{1+t}\right). \end{aligned}$$
  Let L_1 be the line through the points D and P. Then the equation of L_1 is $y-42\sqrt{2}=-\left(\dfrac{1+t}{t}\right)x$. The point of intersection of L_1 and the line $y=x$ is the point E. Therefore,
$$x-42\sqrt{2} = -\left(\frac{1+t}{t}\right)x,\ \text{and so}$$

$$
\begin{aligned}
x &= \frac{42\sqrt{2}\cdot t}{2t+1}. \text{ Thus,}\\
E &= \left(\frac{42\sqrt{2}\cdot t}{2t+1}, \frac{42\sqrt{2}\cdot t}{2t+1}\right) \text{ so that}\\
AE &= \sqrt{2\left(\frac{42\sqrt{2}\cdot t}{2t+1}\right)^2} = \frac{84\cdot t}{2t+1}.
\end{aligned}
$$

Let L_2 be the line through D and Q. Then the equation of L_2 is $y - 42\sqrt{2} = -\left(\dfrac{1}{1+t}\right)x$. Since F is the point of intersection of L_2 and $y = x$, we obtain $x = \dfrac{42\sqrt{2}(t+1)}{t+2}$. Thus,

$$
\begin{aligned}
F &= \left(\frac{42\sqrt{2}(t+1)}{t+2}, \frac{42\sqrt{2}(t+1)}{t+2}\right) \text{ so that}\\
AF &= \frac{84(t+1)}{t+2}.
\end{aligned}
$$

Using the distance formula, one obtains

$$
CF = \sqrt{2\left(42\sqrt{2} - \frac{42\sqrt{2}(t+1)}{t+2}\right)^2} = \frac{84}{t+2}.
$$

As a result,

$$
AE = \frac{84\cdot t}{2t+1},\ AF = \frac{84(t+1)}{t+2},\ \text{and } CF = \frac{84}{t+2}
$$

If $t = 10$, then $AE = 40, AF = 77$, and $CF = 7$. Therefore $EF = AF - AE = 37$, yielding the claimed values. Finally, one checks that for these values all triangles in the figure are defined.

Also solved by Shai Covo, Kiryat-Ono, Israel; Paul M. Harms, North Newton, KS; Boris Rays, Brooklyn, NY; David Stone and John Hawkins (jointly), Statesboro, GA, and the proposer.

- **5117:** *Proposed by Kenneth Korbin, New York, NY*

Find positive acute angles A and B such that

$$
\sin A + \sin B = 2\sin A\cdot\cos B.
$$

Solution by David Stone and John Hawkins (jointly), Statesboro, GA

There are infinitely many solutions, given by

$$A = \sin^{-1}\left(\frac{\sqrt{1-t^2}}{2t-1}\right), \ \mathrm{B} = \cos^{-1} t, \quad \text{where } \frac{4}{5} < t < 1.$$

Here's why.

The given condition is equivalent to

$$2\sin A(2\cos B - 1) = \sin B$$

so we see that $2\cos B - 1 > 0$, that is, $0 < B < \frac{\pi}{3}$.

Solving for $\sin A$, we must have $\sin A = \frac{\sin B}{2\cos B - 1}$, which requires $0 \leq \frac{\sin B}{2\cos B - 1} \leq 1$.

Upon squaring, this is equivalent to

$$\begin{aligned} \sin^2 B &\leq 4\cos^2 B - 4\cos B + 1 \\ 1 - \cos^2 B &\leq 4\cos^2 B - 4\cos B + 1 \\ \cos B &\geq \frac{4}{5}. \end{aligned}$$

So if we choose angle B to make $\cos B \geq \frac{4}{5}$, then we can choose angle A to make $\sin A = \frac{\sin B}{2\cos B - 1}$.

Since cosine is decreasing in the first quadrant, the size condition on $\cos B$ forces $B \leq \cos^{-1}\left(\frac{4}{5}\right) \approx 36.87°$.

In fact, for any t, with $\frac{4}{5} \leq t \leq 1$, we can let $B = \cos^{-1} t$, in which case $\sin B = \sqrt{1-t^2}$, and let $A = \sin^{-1}\left(\frac{\sqrt{1-t^2}}{2t-1}\right)$.

Note that the endpoint "solution" given by $t = 1$ is $A = 0, B = 0$, which we disregard.

Also, the endpoint solution given by $t = \frac{4}{5}$ is $A = \frac{\pi}{2}, B = \cos^{-1}\frac{4}{5}$.

It is worth noting that we thus have a right triangle solution, but it doesn't quite meet the problem's criteria, so we'll disregard this one. Thus, there are infinitely many solutions, given in terms of the parameter t for $\frac{4}{5} < t < 1$.

We also note that one could also say that all solutions are given by $\sin A = \frac{\sin B}{2\cos B - 1}$, where angle B is chosen so that $\cos B > \frac{4}{5}$.

Also solved by Dionne Bailey, Elsie Campbell, and Charles Diminnie (jointly), San Angelo, TX; Michael Brozinsky, Central Islip, NY; Shai Covo, Kiryat-Ono, Israel; Paul M. Harms, North Newton, KS; David E. Manes, Oneonta, NY; Charles McCracken, Dayton, OH; Raúl A. Simón, Santiago, Chile; Taylor University Problem Solving Group; Upland, IN, and the proposer.

- **5118:** *Proposed by David E. Manes, Oneonta, NY*

  Find the value of

$$\sqrt{2011+2007\sqrt{2012+2008\sqrt{2013+2009\sqrt{2014+\cdots}}}}$$

Solution 1 by Shai Covo, Kiryat-Ono, Israel

The value is 2009. More generally, for any integer $n \geq 3$ we have

$$n = \sqrt{(n+2)+(n-2)\sqrt{(n+3)+(n-1)\sqrt{(n+4)+n\sqrt{(n+5)+\cdots}}}}$$

($n = 2009$ corresponds to the original problem.) The claim follows from an iterative application of the identity $n = \sqrt{(n+2)+(n-2)(n+1)}$, as follows:

$$\begin{aligned} n &= \sqrt{(n+2)+(n-2)(n+1)} \\ &= \sqrt{(n+2)+(n-2)\sqrt{(n+3)+(n-1)(n+2)}} \\ &= \sqrt{(n+2)+(n-2)\sqrt{(n+3)+(n-1)\sqrt{(n+4)+n(n+3)}}} \\ &= \cdots. \end{aligned}$$

Solution 2 by Taylor University Problem Solving Group, Upland, IN

We use Ramanujan's nested radical approach. Beginning with

$$(x+n+a)^2 = x^2+n^2+a^2+2ax+2nx+2an,$$

we see that

$$\begin{aligned} x+n+a &= \sqrt{x^2+n^2+a^2+2ax+2nx+2an} \\ &= \sqrt{ax+n^2+a^2+2an+x\,(x+2n+a)} \\ &= \sqrt{ax+(n+a)^2+x\,(x+2n+a)}. \end{aligned}$$

However, the $(x+2n+a)$ term on the right is basically of the same form as the left (with n replaced by $2n$). We can make the corresponding substitution, and continue this process indefinitely, until we are left with $x+n+a =$

$$\sqrt{ax+(n+a)^2+x\sqrt{a(x+n)+(n+a)^2+(x+n)\sqrt{a\,(x+2n)+(n+a)^2+(x+2n)\sqrt{\cdots}}}}$$

Substituting in $x = 2007$, $n = a = 1$ produces

$$\begin{aligned} 2009 &= \sqrt{2007+4+2007\sqrt{2008+4+2008\sqrt{2009+4+2009\sqrt{\cdots}}}} \\ &= \sqrt{2011+2007\sqrt{2012+2008\sqrt{2013+2009\sqrt{\cdots}}}}. \end{aligned}$$

Hence, the value is 2009.

Also solved by Scott H. Brown, Auburn University, Montgomery, AL; G. C. Greubel, Newport News, VA; Paul M. Harms, North Newton, KS: Kenneth Korbin, NY, NY; Charles McCracken, Dayton, OH; Paolo Perfetti, Department of Mathematics, University of Rome, Italy; Boris Rays, Brooklyn, NY; David Stone and John Hawkins (jointly), Stateboro GA, and the proposer.

- **5119:** *Proposed by Isabel Díaz-Iriberri and José Luis Díaz-Barrero, Barcelona, Spain*

Let n be a non-negative integer. Prove that

$$2+\frac{1}{2^{n+1}}\prod_{k=0}^{n}\csc\left(\frac{1}{F_k}\right)<F_{n+1}$$

where F_n is the n^{th} Fermat number defined by $F_n=2^{2^n}+1$ for all $n\geq 0$.

Solution by Charles R. Diminnie, San Angelo, TX

To begin, we note that for $x\in\left(0,\dfrac{\pi}{3}\right)$, $\cos x$ is decreasing and the Mean Value Theorem for Derivatives implies that there is a point $c_x\in(0,x)$ such that

$$\begin{aligned}\sin x &= \sin x-\sin 0\\ &= \cos c_x\,(x-0)\\ &> \cos\frac{\pi}{3}\cdot x\\ &= \frac{x}{2}.\end{aligned}$$

As a result, when $x\in\left(0,\dfrac{\pi}{3}\right)$,

$$x\csc x<2.$$

Since $F_n\geq F_0=3$ for all $n\geq 0$, it follows that $0<\dfrac{1}{F_n}\leq\dfrac{1}{3}<\dfrac{\pi}{3}$ and hence,

$$\begin{aligned}\frac{1}{F_n}\csc\left(\frac{1}{F_n}\right) &< 2,\ \text{or}\\ \csc\left(\frac{1}{F_n}\right) &< 2F_n \qquad (1)\end{aligned}$$

Let $P(n)$ be the statement

$$\prod_{k=0}^{n}\csc\left(\frac{1}{F_k}\right)<2^{n+1}\left(F_{n+1}-2\right) \qquad (2)$$

By (1),

$$\csc\left(\frac{1}{F_0}\right)<2F_0=2\cdot 3=2\left(F_1-2\right)$$

and $P(0)$ is true. If $P(n)$ is true for some $n \geq 0$, then by (1),

$$\begin{aligned}
\prod_{k=0}^{n+1} \csc\left(\frac{1}{F_k}\right) &= \csc\left(\frac{1}{F_{n+1}}\right) \prod_{k=0}^{n} \csc\left(\frac{1}{F_k}\right) \\
&< \csc\left(\frac{1}{F_{n+1}}\right) \cdot 2^{n+1}\left(F_{n+1}-2\right) \\
&< 2F_{n+1} \cdot 2^{n+1}\left(F_{n+1}-2\right) \\
&= 2^{n+2}\left(2^{2^{n+1}}+1\right)\left(2^{2^{n+1}}-1\right) \\
&= 2^{n+2}\left(2^{2^{n+2}}-1\right) \\
&= 2^{n+2}\left(F_{n+2}-2\right)
\end{aligned}$$

and $P(n+1)$ follows. By Mathematical Induction, $P(n)$ is true for all $n \geq 0$.

Since (2) is equivalent to the given inequality, the proof is complete.

Also solved by Shai Covo, Kiryat-Ono, Israel; Bruno Salgueiro Fanego, Viveiro, Spain; David Stone and John Hawkins (jointly), Statesboro, GA, and the proposers.

- **5120:** *Proposed by José Luis Díaz-Barrero, Barcelona, Spain*

Calculate

$$\lim_{n\to\infty} \frac{1}{2^n} \sum_{k=0}^{n} (-1)^k \binom{n}{k} \log\left(\frac{2n-k}{2n+k}\right).$$

Solution 1 by Ovidiu Furdui, Cluj, Romania

The limit equals 0. More generally, we prove that if $f : [0,1] \to \Re$ is a continuous function then

$$\lim_{n\to\infty} \frac{1}{2^n} \sum_{k=0}^{n} (-1)^k \binom{n}{k} f\left(\frac{k}{n}\right) = 0.$$

Before we give the solution of the problem we collect the following equality from **[1]** (Formula 0.154(3), p.4): If $p \geq 0$ is a nonnegative integer, then the following equality holds

$$\sum_{k=0}^{n} (-1)^k \binom{n}{k} k^p = 0. \qquad (1)$$

Now we are ready to solve the problem. First we note that for a polynomial $P(x) = \sum_{j=0}^{m} a_j x^j$ we have, based on (1), that

$$\frac{1}{2^n} \sum_{k=0}^{n} (-1)^k \binom{n}{k} P\left(\frac{k}{n}\right) = \sum_{j=0}^{m} \frac{a_j}{n^j} \cdot \frac{1}{2^n} \left(\sum_{k=0}^{n} (-1)^k \binom{n}{k} k^j\right) = 0. \qquad (2)$$

Let $\epsilon > 0$ and let P_ϵ be the polynomial that uniformly approximates f, i.e. $|f(x) - P_\epsilon(x)| < \epsilon$ for all $x \in [0,1]$. We have, based on (2), that

$\frac{1}{2^n}\sum_{k=0}^{n}(-1)^k\binom{n}{k}P_\epsilon\left(\frac{k}{n}\right)=0$. Thus,

$$\begin{aligned}\left|\frac{1}{2^n}\sum_{k=0}^{n}(-1)^k\binom{n}{k}f\left(\frac{k}{n}\right)\right| &= \left|\frac{1}{2^n}\sum_{k=0}^{n}(-1)^k\binom{n}{k}\left(f\left(\frac{k}{n}\right)-P_\epsilon\left(\frac{k}{n}\right)\right)\right| \\ &\leq \frac{1}{2^n}\sum_{k=0}^{n}\binom{n}{k}\left|f\left(\frac{k}{n}\right)-P_\epsilon\left(\frac{k}{n}\right)\right| \\ &\leq \frac{\epsilon}{2^n}\sum_{k=0}^{n}\binom{n}{k} \\ &= \epsilon.\end{aligned}$$

Thus, the limit is 0 and the problem is solved.

[**1**] I.S. Gradshteyn and I.M. Ryzhik, Table of Integrals, Series, and Products, Sixth Edition, Alan Jeffrey, Editor, Daniel Zwillinger, Associate Editor, 2000.

Solution 2 by Shai Covo, Kiryat-Ono, Israel

We will show that

$$\lim_{n\to\infty}\frac{1}{2^n}\sum_{k=0}^{n}(-1)^k\binom{n}{k}\log\left(\frac{2n-k}{2n+k}\right)=0. \qquad (1)$$

(The log function in (1) has no significant role in the analysis below, we could replace it by any other continuous function.)

The lemma below follows straightforwadly from the Central Limit Theorem (CLT). We recall that, according to the CLT, if $X_1, X_2, \ldots$ is a sequence of independent and identically distributed (i.i.d) random variables with expectation μ and variance σ^2, then

$$P\left(a<\frac{X_1+\cdots+X_n-n\mu}{\sigma\sqrt{n}}\leq b\right)\to\Phi(b)-\Phi(a) \qquad (2)$$

as $n\to\infty$, for any $a,b\in\Re$ with $a<b$ where Φ is the distribution function of the Normal $(0,1)$ distribution (i.e., $\Phi(x)=(2\pi)^{-1/2}\int_{-\infty}^{x}e^{-\mu^2/2}du$).

Lemma: For any $\epsilon>0$, there exists an $r>0$ such that

$$\frac{1}{2^n}\sum_{\substack{0\leq k\leq n/2-r\sqrt{n}\\ n/2+r\sqrt{n}<k\leq n}}\binom{n}{k}<\epsilon \qquad (3)$$

for all n sufficiently large.

Proof: Fix $\epsilon>0$. Choose $r>0$ sufficiently large so that $\Phi(2r)-\Phi(-2r)>1-\epsilon$. Let $X_1, X_2\ldots$ be a sequence of i.i.d. variables with $P(X_i=0)=P(X_i=1)=1/2$. Put $Y_n=\sum_{i=1}^{n}X_i$. Thus Y_n has a binomial $(n,1/2)$ distribution. The X_i's have expectation $\mu=1/2$ and variance $\sigma^2=1/4$. Hence by (2) (with $a=-2r$ and $b=2r$),

$$P(n/2-r\sqrt{n}<Y_n\leq n/2+r\sqrt{n})>1-\epsilon$$

for all n sufficiently large. In turn, by taking complements, we conclude (3), since the distribution of Y_n is given by $P(Y_n = k) = \frac{1}{2^n}\binom{n}{k}, k = 0, \ldots, n.$

It follows from the lemma and the fact that $\left|(-1)^k \log\left(\frac{2n-k}{2n+k}\right)\right|$ is bounded uniformly in k (say, by 2) that (1) will be proved if we show that

$$\lim_{n\to\infty} \frac{1}{2^n} \sum_{n/2 - r\sqrt{n} < k < n/2 + r\sqrt{n}} (-1)^k \binom{n}{k} \log\left(\frac{2n-k}{2n+k}\right) = 0 \qquad (4)$$

for any fixed $r > 0$. This is shown as follows. We first write

$$\begin{aligned} &\left|(-1)^k \binom{n}{k} \log\left(\frac{2n-k}{2n+k}\right) + (-1)^k \binom{n}{k+1} \log\left(\frac{2n-(k+1)}{2n+(k+1)}\right)\right| \\ = \quad &\binom{n}{k}\left|\log\left(\frac{2n-k}{2n+k}\right) - \frac{n-k}{k+1}\log\left(\frac{2n-(k+1)}{2n+(k+1)}\right)\right|. \qquad (5) \end{aligned}$$

Clearly, the expression multipying $\binom{n}{k}$ on the right of the equality in (5) can be made arbitrarily small uniformly in $k \in [n/2 - r\sqrt{n},\ n/2 + r\sqrt{n}]$, where $r > 0$ is fixed, by choosing n sufficiently large. Then, in view of the triangle inequality, (4) follows from $\frac{1}{2^n}\sum_{k=0}^{n}\binom{n}{k}\epsilon = \epsilon$ (where $\epsilon > 0$ is arbitrarily small) and $\binom{n}{k}\Big/2^n \overset{\text{unif.}}{\longrightarrow} 0$ (to be used if the sum in (4) consists of an odd number of terms). The desired result (1) is thus proved.

Also proved by Boris Rays, Brooklyn, NY and the proposer.

5121: *Proposed by Tom Leong, Scotrun, PA*

Let n, k and r be positive integers. It is easy to show that

$$\sum_{n_1+n_2+\cdots+n_r=n} \binom{n_1}{k}\binom{n_2}{k}\cdots\binom{n_r}{k} = \binom{n+r-1}{kr+r-1}, \quad n_1, n_2, \cdots n_r \in N$$

using generating functions. Give a combinatorial argument that proves this identity.

Solution 1 by Shai Covo, Kiryat-Ono, Israel

Suppose we have n identical boxes and kr ($\leq n$) identical balls. The stated equality is trivial if $r = 1$, hence we can assume $r > 1$.

We begin with the left-hand side of the stated equality. Assuming $n_1, \ldots, n_r \geq k$, it gives the number of ways to divide the n boxes into r groups–the ith group having $n_i \geq k$ elements–and put exactly k balls in each group.

As for the right-hand side, suppose that in addition to the n boxes and the kr balls we have $r-1$ separators. This gives rise to an $(n+r-1)$-tuple of boxes and separators. We denote this tuple by M. We identify a sequence $(i_1, i_2, \ldots, i_{kr+r-1})$ such that $1 \le i_1 < i_2 < \cdots < i_{kr+r-1} \le n+r-1$ with the following arrangement: the i_jth $(j = 1, \ldots, kr+r-1)$ element of M is a separator if j is a multiple of $k+1$ and a box containing a ball otherwise. (The remaining $n-kr$ elements are empty boxes.) We thus conclude that $\binom{n+r-1}{kr+r-1}$ gives the number of ways to place $r-1$ separators between the n boxes and kr balls into the boxes, such that each of the resulting r groups contains exactly k balls. This establishes the equality of the left-and right-hand sides.

Solution 2 by the proposer

Both sides count the number of possible ways to arrange $kr+r-1$ green balls and $n-kr$ red balls in a row. This is clearly true for the right side. In the left side, note that any term in the sum with $n_i < k$ for some i is equal to zero; so we may assume $n_i \ge k$ for all i. For each *composition* $n_1 + \cdots + n_r = n$ of n, consider the row of n red and $r-1$ green balls arranged as

$$\underbrace{RR\cdots R}_{n_1 \text{ balls}} G \underbrace{RR\cdots R}_{n_2 \text{ balls}} G \underbrace{RR\cdots R}_{n_3 \text{ balls}} G \cdots G \underbrace{RR\cdots R}_{n_{r-1} \text{ balls}} G \underbrace{RR\cdots R}_{n_r \text{ balls}}$$

From each block of red balls, choose k of them and paint them green. The number of ways to do this is $\binom{n_1}{k}\binom{n_2}{k}\cdots\binom{n_r}{k}$. This results in a row consisting of $kr+r-1$ green balls and $n-kr$ red balls. Conversely, in any row consisting of $kr+r-1$ green balls and $n-kr$ red balls, we can determine a unique composition $n_1 + n_2 + \cdots + n_r = n$ of n by reversing the process.