Problems

Ted Eisenberg, Section Editor

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This section of the Journal offers readers an opportunity to exchange interesting mathematical problems and solutions. Please send them to Ted Eisenberg, Department of Mathematics, Ben-Gurion University, Beer-Sheva, Israel or fax to: 972-86-477-648. Questions concerning proposals and/or solutions can be sent e-mail to <eisenbt@013.net>. Solutions to previously stated problems can be seen at <http://www.ssma.org/publications>.

Solutions to the problems stated in this issue should be posted before February 15, 2019

5517: *Proposed by Kenneth Korbin, New York, NY*

Find positive integers (a, b, c) such that
$\arccos\left(\dfrac{a}{1331}\right) = \arccos\left(\dfrac{b}{1331}\right) + \arccos\left(\dfrac{c}{1331}\right)$ with $a < b < c$.

5518: *Proposed by Roger Izard, Dallas, TX*

Let triangle PQR be equilateral and let it intersect another triangle ABC at points U, U', W, W', V, V' such that WU', UV', VW' are equal in length, and triangles $AU'W, BV'U, CW'V$ are equal in area (see Figure 1). Show that triangle ABC must then also be equilateral

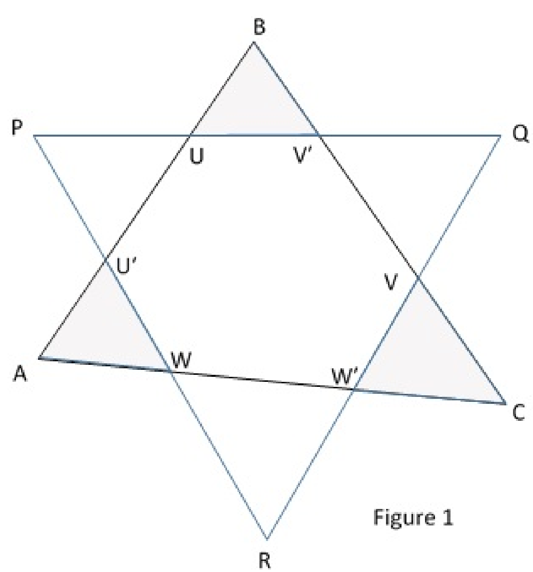

Figure 1

5519: *Proposed by Titu Zvonaru, Comănesti, Romania*

Let a, b, c be positive real numbers. Prove that

$$\frac{a^2}{b^2} + \frac{b^2}{c^2} + \frac{c^2}{a^2} + \frac{2abc}{a^3 + b^3 + c^3} \geq \frac{11}{3}.$$

5520: *Proposed by Raquel León (student) and Angel Plaza, University of Las Palmas de Gran Canaria, Spain*

Let n be a positive integer. Prove that

$$\sum_{k=0}^{2n} \binom{2n+k}{k} \binom{2n}{k} \frac{(-1)^k}{2^k} \frac{1}{k+1} = 0.$$

5521: *Proposed by José Luis Díaz-Barrero, Barcelona Tech, Barcelona, Spain*

Let $a > 0$ be a real number. If f is an odd non-constant real function having second derivative in the interval $[-a, a]$ and $f'(-a) = f'(a) = 0$, then prove that there exists a point $c \in (-a, a)$ such that

$$\frac{1}{2} f''(c) \geq \frac{|f(a)|}{a^2}$$

5522: *Proposed by Ovidiu Furdui and Cornel Vălean from Technical University of Cluj-Napoca, Cluj-Napoca, Romania and Timiş, Romania, respectively*

Calculate

$$\int_0^1 \int_0^1 \frac{\log(1-x) - \log(1-y)}{x-y} dxdy.$$

Solutions

5499: *Proposed by Kenneth Korbin, New York, NY*

Given a triangle with sides $(21, 23, 40)$. The sum of these digits is $2 + 1 + 2 + 3 + 4 + 0 = 12$. Find primitive pythagorean triples in which the sum of the digits is 12 or less.

Solution 1 by Anthony J. Bevelacqua, University of North Dakota, Grand Forks, ND

The two best-known primitive pythagorean triples $(3, 4, 5)$ and $(5, 12, 13)$ have this property. We will give two infinite families of such triples.

Recall that $a = s^2 - t^2$, $b = 2st$, $c = s^2 + t^2$ is a primitive pythagorean triple for any $s > t \geq 1$ with $\gcd(s, t) = 1$ and s and t of opposite parity.

1. For any $n \geq 1$ let $s = 10^n + 1$ and $t = 10^n$. Then

$$\begin{array}{rcccl} a & = & s^2 - t^2 & = & 2 \cdot 10^n + 1 \\ b & = & 2st & = & 2 \cdot 10^{2n} + 2 \cdot 10^n \\ c & = & s^2 + t^2 & = & 2 \cdot 10^{2n} + 2 \cdot 10^n + 1 \end{array}$$

   is a primitive pythagorean triple. The sum of the digits in (a, b, c) is

$$2 + 1 + 2 + 2 + 2 + 2 + 1 = 12.$$

2. For any $n \geq 1$ let $s = 10^{2n} + 1$ and $t = 10^n$. Then

$$\begin{array}{rcccl} a & = & s^2 - t^2 & = & 10^{4n} + 10^{2n} + 1 \\ b & = & 2st & = & 2 \cdot 10^{3n} + 2 \cdot 10^n \\ c & = & s^2 + t^2 & = & 10^{4n} + 3 \cdot 10^{2n} + 1 \end{array}$$

   is a primitive pythagorean triple. The sum of the digits in (a, b, c) is

$$1 + 1 + 1 + 2 + 2 + 1 + 3 + 1 = 12.$$

Solution 2 by Dionne Bailey, Elsie Campbell, Charles Diminnie, and Trey Smith, Angelo State University, San Angelo, TX

To begin, we note the well-known result that (a, b, c) is a primitive Pythagorean triple, with $a^2 + b^2 = c^2$ and a odd if and only if $a = m^2 - n^2$, $b = 2mn$, and $c = m^2 + n^2$ for some positive integers m and n such that $m > n$, $\gcd(m, n) = 1$, and m and n have opposite parity.

We can provide an infinite family of primitive Pythagorean triples for which the sum of the digits is 12 by choosing $m = 10^k + 1$ and $n = 10^k$ with $k \geq 0$. Then,

$$a_k = \left(10^k + 1\right)^2 - 10^{2k} = 2 \times 10^k + 1,$$

$$b_k = 2\left(10^k + 1\right)\left(10^k\right) = 2 \times 10^{2k} + 2 \times 10^k,$$

and

$$c_k = \left(10^k + 1\right)^2 + 10^{2k} = 2 \times 10^{2k} + 2 \times 10^k + 1$$

for $k \geq 0$. As noted above, $(a_k . b_k, c_k)$ is a primitive Pythagorean triple for each $k \geq 0$. Further, in each case, the sum of the non-zero digits for a_k, b_k, and c_k is $(2 + 1) + (2 + 2) + (2 + 2 + 1) = 12$. In particular, when $k = 0$, we have $(a_0, b_0, c_0) = (3, 4, 5)$, the best known primitive Pythagorean triple.

Another example is $(a, b, c) = (5, 12, 13)$. However, we haven't been able to generalize this in a manner similar to that shown above. Also, we haven't found any other examples of primitive Pythagorean triples (a, b, c) for which the sum of the digits of a, b, and c is 12 or less.

Editor's comment : **David Stone and John Hawkins, both of Georgia Southern University in Stateboro, GA** stated that a computer search revealed no triples with total digit sum < 12. In each triple with total digit sum 12, the 12 was achieved as

3+4+5. They also found no triples with total digit sum 13 or 14. They went on to find the above mentioned infinite class with digit sum15 and ended their solution with the comment: "We do not know whether there are other triples with total digit sum 12. Note that $x+y+z = 2ab + \left(b^2 - a^2\right) + \left(b^2 + a^2\right) = 2b(ab)$. For any integer w we know that $w = \text{Digitsum}(w) \mod 3$. Thus Digitsum (x)+ Digitsum (y)+ Digitsum $(z) = x+y+z = 2b(a+b) \mod 3$. So if the total digit sum is 12, then $b = 0 \mod 3$ or $a+b = 0 \mod 3$. That is, there are restrictions on the generators a and b."

Also solved by Ed Gray, Highland Beach, FL; David E. Manes, Oneonta, NY; Ioannis D. Sfikas, National and Kapodistrian University of Athens, Greece; Albert Stadler, Herrliberg, Switzerland, and the proposer.

5500: *Proposed by Moshe Stupel, "Shaanan" Academic College of Education and Gordon Academic College of Education, and Avi Sigler, "Shaanan" Academic College of Education, Haifa, Israel*

Without the use of a calculator, show that: $8\sin 20^\circ \cdot \sin 40^\circ \cdot \sin 60^\circ \cdot \sin 80^\circ = \dfrac{3}{2}$.

Solution 1 by David A. Huckaby, Angelo State University, San Angelo, TX

$$\begin{aligned}
&8\sin 20^\circ \cdot \sin 40^\circ \cdot \sin 60^\circ \cdot \sin 80^\circ \\
&\quad = 8\sin(30^\circ - 10^\circ) \cdot \sin(30^\circ + 10^\circ) \cdot \sin 60^\circ \cdot \sin(90^\circ - 10^\circ) \\
&\quad = 8[\sin 30^\circ \cdot \cos 10^\circ - \cos 30^\circ \cdot \sin 10^\circ] \cdot [\sin 30^\circ \cdot \cos 10^\circ + \cos 30^\circ \cdot \sin 10^\circ] \\
&\qquad\quad \cdot \frac{\sqrt{3}}{2} \cdot [\sin 90^\circ \cdot \cos 10^\circ - \cos 90^\circ \cdot \sin 10^\circ] \\
&\quad = 8\left[\frac{1}{2} \cdot \cos 10^\circ - \frac{\sqrt{3}}{2} \cdot \sin 10^\circ\right] \cdot \left[\frac{1}{2} \cdot \cos 10^\circ + \frac{\sqrt{3}}{2} \cdot \sin 10^\circ\right] \cdot \frac{\sqrt{3}}{2} \cdot [\cos 10^\circ] \\
&\quad = \sqrt{3}\cos 10^\circ \cdot \left[\cos 10^\circ - \sqrt{3}\sin 10^\circ\right] \cdot \left[\cos 10^\circ + \sqrt{3}\sin 10^\circ\right] \\
&\quad = \sqrt{3}\cos 10^\circ \cdot \left[\cos^2 10^\circ - 3\sin^2 10^\circ\right] \\
&\quad = \sqrt{3}\left[\cos^3 10^\circ - 3\sin^2 10^\circ \cdot \cos 10^\circ\right] \\
&\quad = \sqrt{3}\cos 30^\circ \\
&\quad = \sqrt{3} \cdot \frac{\sqrt{3}}{2} \\
&\quad = \frac{3}{2}
\end{aligned}$$

Solution 2 by Cartesian Gains Student Problem Solving Group, Mountain Lakes High School, Mountain Lakes, NJ

We use the well known formula: $\sin\theta = \dfrac{e^{i\theta} - e^{-i\theta}}{2i}$.

Converting to radians, we rewrite the left side of the equation as

$$8 \cdot \left(\frac{1}{2i}\right)^4 \cdot \left(e^{i\pi/9} - e^{-i\pi/9}\right) \cdot \left(e^{i2\pi/9} - e^{-i2\pi/9}\right) \cdot \left(e^{i3\pi/9} - e^{-i3\pi/9}\right) \cdot \left(e^{i4\pi/9} - e^{-i4\pi/9}\right).$$

Expanding gives:

$$\frac{1}{2}\cdot\left(e^{i(10\pi/9)}+e^{-i(10\pi/9)}-e^{i(8\pi/9)}-e^{-i(8\pi/9)}-e^{i(6\pi/9)}-e^{-i(6\pi/9)}+2\right) \tag{1}$$

We use the fact that on the unit circle $e^{i(10\pi/9)}$ represents the same complex number as $e^{-i(8\pi/9)}$. Similarly, $e^{i(8\pi/9)}=e^{-i(10\pi/9)}$. These terms cancel out in our equation.

Additionally,

$$-\left(e^{i(6\pi/9)}+e^{-i(6\pi/9)}\right)=-\left(\cos\frac{6\pi}{9}+i\sin\frac{6\pi}{9}+\cos\frac{-6\pi}{9}+i\sin\frac{-6\pi}{9}\right)=-2\cos(6\pi/9)=1.$$

Therefore, equation (1) reduces to: $\dfrac{1}{2}(1+2)=\dfrac{3}{2}$.

Editor′s comments: **Albert Stadler of Herrliberg, Switzerland** and several other solvers, noticed that this problem is a special case of problem 5497, which asked us to find a closed form of

$$\prod_{k=1}^{n-1}2\sin\left(\frac{k\pi}{n}\right).$$

He showed that

$$\prod_{k=1}^{n-1}2\sin\left(\frac{k\pi}{n}\right)=\lim_{x}1\frac{x^n-1}{x-1}=n.$$

By symmetry $\sin\left(\dfrac{k\pi}{n}\right)=\sin\left(\dfrac{n-k)\pi}{n}\right)$, so if n is odd then

$$\prod_{k=1}^{\frac{n-1}{2}}2\sin\left(\frac{k\pi}{n}\right)=\sqrt{n}.$$

So problem is 5500 is the special case with $n=9$.

Yagub Alyiev of ADA University in Baku, Azerbaijan, mentor to the two students listed below from his university who solved the problem, sent two web addresses wherein animated solutions can be found. See:

https://www.youtube.com/watch?v=Tc58b2AGFf4 (and)

https://www.youtube.com/watch?v=zAiXPhPvWpct=187s.

Also solved by Arkady Alt; San Jose, CA; Michel Bataille, Rouen, France; Brian D. Beasley (two solutions), Presbyterian College, Clinton, SC; Anthony J. Bevelacqua, University of North Dakota, Grand Forks, ND; Scott H. Brown, Montgomery, AL; Michael Brozinsky, Central Islip, NY; Elsie Campbell, Dionne T. Bailey, Charles Diminnie, and Trey Smith, Angelo State University, San Angelo, TX; : Michael C. Faleski, Delta College, University Center, MI; Bruno Salgueiro Fango, Viveiro, Spain; Ed Gray, Highland Beach, FL; Vagif Hamzayev(student), ADA University, Baku, Azerbaijan; Paul M. Harms, North Newton, KS; Kee-Wai Lau, Hong Kong, China; Carl Libis, Columbia Southern University, Orange Beach, AL;

David E. Manes, Oneonta, NY; Kamal Mustafayev (student), ADA University, Baku, Azerbaijan; Pedro H.O. Pantoja, Natal/RN, Brazil; Paolo Perfetti, Department of Mathematics, Tor Vergata University, Rome, Italy; Ioannis D. Sfikas (two solutions), National and Kapodistrian University of Athens, Greece; Digby Smith, Mount Royal University, Calgary, Canada; Albert Stadler of Herrliberg, Switzerland; Neculai Stanciu, "George Emil Palade" School Buzău, Romania and Titu Zvonaru, Comănesti, Romania; David Stone and John Hawkins of Georgia Southern University, Statesboro, GA; Daniel Văcaru, Pitesti, Romania, and the proposers.

5501: *Proposed by D.M. Bătinetu-Giurgiu, Bucharest, Romania, Neculai Stanciu, "George Emil Palade" School Buzău, Romania and Titu Zvonaru, Comănesti, Romania*

Determine all real numbers a, b, x, y that simultaneously satisfy the following relations:

$$\begin{cases} (1) \quad ax + by = 5 \\ (2) \quad ax^2 + by^2 = 9 \\ (3) \quad ax^3 + by^3 = 17 \\ (4) \quad ax^4 + by^4 = 33. \end{cases}$$

Solution 1 by Stanley Rabinowitz, Chelmsford, MA

(1) $ax + by = 5$.
(2) $ax^2 + by^2 = 9$.
(3) $ax^3 + by^3 = 17$.
(4) $ax^4 + by^4 = 33$.
(5) $ax = 5 - by$, by (1).
(6) $(5 - by)xy + by^3 = 9y$, by (2) and (5).
(7) $ax^2 = 9 - by^2$, by (2).
(8) $(9 - by^2)x + by^3 = 17$, by (3) and (7).
Subtracting (8) from (6) gives
(9) $5xy - 9x = 9y - 17$.
(10) $ax^3 = 17by^3$, by (3).
(11) $(17 - by^3)x + by^4 = 33$, by (4) and (10).
Multiplying (8) by y and subtracting (11) yields
(12) $9xy - 17x = 17y - 33$.
Subtracting 5 times (12) from 9 times (9) and dividing the result by 4 gives (13) $x = -y + 3$.
Substituting this value of x into (9) and simplifying yields: $-5(y - 1)(y - 2) = 0$.
Therefore, $y = 1$ or 2.

Suppose $y = 1$. Then, $x = 2$, by (13). Thus, $2a + b = 5$ and $4a + b = 9$, by (1) and (2). Hence, $a = 2$ and $b = 1$. That is, $(x, y, a, b) = (2, 1, 2, 1)$.

Similarly, if $y = 2$, then $(x, y, a, b) = (1, 2, 1, 2)$.

Note that this result holds in any commutative ring with unity, which has no zero divisors and $5 \neq 0$.

Solution 2 by David E. Manes, Oneonta, NY

Writing $5 = 2^2 + 1$, $9 = 2^3 + 1$, $17 = 2^4 + 1$ and $33 = 2^5 + 1$, one notes that two of the solutions (a, b, x, y) for the system of equations are $(1, 2, 1, 2)$ and $(2, 1, 2, 1)$. We will show that these are the only solutions. Let A be the augmented 4×3 matrix for the system of equations where a and b are regarded as the unknowns and the powers of x and y are regarded as the coefficients. Then

$$A = \begin{bmatrix} x & y & 5 \\ x^2 & y^2 & 9 \\ x^3 & y^3 & 17 \\ x^4 & y^4 & 33 \end{bmatrix}.$$

Row-reducing A, we find that it is row-equivalent to the matrix R given by

$$R = \begin{bmatrix} 1 & 0 & \frac{5y-9}{x(y-x)} \\ 0 & 1 & \frac{9-5x}{y(y-x)} \\ 0 & 0 & 17 - 9y - 9x + 5xy \\ 0 & 0 & 33 - 9(x^2 + y^2 + xy) + 5xy(y + x) \end{bmatrix}.$$

If $x = 1$ and $y = 2$, then the two expressions $17 - 9y - 9x + 5xy$ and $33 - 9(x^2 + y^2 + xy) + 5xy(y + x)$ both equal 0. Therefore, $a = 1$ and $b = 2$ since $\frac{5y-9}{x(y-x)} = 1$ and $\frac{9-5x}{y(y-x)} = 2$ when $x = 1$ and $y = 2$. If $x = 2$ and $y = 1$, then $17 - 9x - 9y + 5xy = 33 - 9(x^2 + y^2 + xy) + 5xy(y + x) = 0$ so that $a = 2$ and $b = 1$. Working with residues modulo 3, one finds that the equation $17 - 9y - 9x + 5xy \equiv 0 \pmod 3$ if and only if $x \equiv 1 \pmod 3$ and $y \equiv 2 \pmod 3$ or $x \equiv 2 \pmod 3$ and $y \equiv 1 \pmod 3$. Furthermore, these residues have to be least residues since otherwise, the residues can be made to satisfy the first equation in the system, but not the second.

Also solved by Arkady Alt; San Jose, CA; Hatef Arshagi, Guilford Technical Community College, Jamestown, NC; Michel Bataille, Rouen, France; Brian D. Beasley, Presbyterian College, Clinton, SC; Anthony Bevelacqua, University of North Dakota, Grand Forks, ND; Cartesian Gains Student Problem Solving Group, Mountain Lakes High School, Mountain Lakes, NJ; Dionne Bailey, Elsie Campbell, Charles Diminnie, and Trey Smith, Angelo State University, San Angelo, TX; Ed Gray, Highland Beach,FL; Paul M. Harms, North Newton, KS; Kee-Wai Lau, Hong Kong, China; Perfetti Paolo, Department of Mathematics, Tor Vergata University, Rome, Italy; Ioannis D. Sfikas (two solutions), National and Kapodistrian University of Athens, Greece; Digby Smith, Mount Royal University, Calgary, Canada; Albert Stadler of Herrliberg, Switzerland; David Stone and John Hawkins,

Georgia Southern University, Statesboro, GA; Daniel Văcaru, Pitesti, Romania, and the proposers.

5502: *Proposed by Daniel Sitaru, "Theodor Costescu" National Economic College, Drobeta Turnu-Severin, Mehedinti, Romania*

Prove that if $a, b, c > 0$ and $a+b+c = e$ then

$$e^{ac^e} \cdot e^{ba^e} \cdot e^{cb^e} > e^e \cdot a^{be^2} \cdot b^{ce^2} \cdot c^{ae^2}.$$

Here, $e = \lim_{n\to\infty} \left(1+\frac{1}{n}\right)^n$

Solution 1 by Paolo Perfetti, Department of Mathematics, Tor Vergata University, Rome, Italy

The inequality is equivalent to

$$ac^e + ba^e + cb^e > e + be^2 \ln a + ce^2 \ln b + ae^2 \ln c$$

that is

$$a(c^e - e^2 \ln c) + b(a^e - e^2 \ln a) + c(b^e - e^2 \ln b) > e$$

Let $f(x) = x^e - e^2 \ln x$.

$$f''(x) = e(e-1)x^{e-2} + \frac{e^2}{x^2} > 0$$

Thus by Jensens's inequality

$$e \sum_{\text{cyc}} \frac{a}{e}(c^e - e^2 \ln c) \geq e\left[\left(\frac{a+b+c}{e}\right)^e - a^2 \ln \frac{a+b+c}{e}\right] = e$$

Solution 2 by Moti Levy, Rehovot, Israel

The function $\ln x$ is monotone increasing, then by applying log function on both sides of the inequality, we get

$$ac^e + ba^e + cb^e > e + be^2 \ln a + ce^2 \ln b + ae^2 \ln c, \tag{1}$$

or

$$\frac{a}{e}c^e + \frac{b}{e}a^e + \frac{c}{e}b^e > 1 + e^2\left(\frac{b}{e}\ln a + \frac{c}{e}\ln b + \frac{a}{e}\ln c\right). \tag{2}$$

The function $\ln x$ is concave, hence

$$\ln\left(\frac{ab+bc+ca}{e}\right) \geq \frac{b}{e}\ln a + \frac{c}{e}\ln b + \frac{a}{e}\ln c. \tag{3}$$

Thus we get for the right hand side of inequality (2) :

$$1 - e^2 + e^2 \ln(ab+bc+ca) \geq 1 + e^2\left(\frac{b}{e}\ln a + \frac{c}{e}\ln b + \frac{a}{e}\ln c\right). \tag{4}$$

The function x^e is convex, hence we get for the left hand side of inequality (2):

$$\frac{a}{e}c^e + \frac{b}{e}a^e + \frac{c}{e}b^e \geq \left(\frac{ab+bc+ca}{e}\right)^e. \tag{5}$$

By (4) and (5), to finish the solution, we have to show that

$$\left(\frac{ab+bc+ca}{e}\right)^e > 1-e^2+e^2\ln(ab+bc+ca). \tag{6}$$

Let us denote

$$x := (ab+bc+ca)^e. \tag{7}$$

Since $ab+bc+ca \le \frac{e^2}{3}$, then

$$0 < x \le \left(\frac{e^2}{3}\right)^e. \tag{8}$$

Setting (7) in (6), we need to show that

$$\frac{x}{e^e} > 1-e^2+e\ln x, \text{ for } \ 0 < x \le \left(\frac{e^2}{3}\right)^e,$$

or that

$$f(x) := x - e^{1+e}\ln x + e^e\left(e^2-1\right) > 0, \text{ for } \ 0 < x \le \left(\frac{e^2}{3}\right)^e. \tag{9}$$

One can easily check that $f'(x) = 1 - \frac{e^{1+e}}{x} < 0$ for $0 < x \le \left(\frac{e^2}{3}\right)^e$. Hence, $f(x)$ is monotone decreasing function for $0 < x \le \left(\frac{e^2}{3}\right)^e$. Moreover, $\lim_{x\to 0} f(x) = +\infty$ and $f\left(\left(\frac{e^2}{3}\right)^e\right) = \left(\frac{e^2}{3}\right)^e - e^{1+e}\left(\ln\left(\frac{e^2}{3}\right)^e\right) + e^e\left(e^2-1\right) \cong 7.4789 > 0$. These and the monotonicity of $f(x)$ imply that $x - e^{1+e}\ln x + e^e\left(e^2-1\right) > 0$, for $0 < x \le \left(\frac{e^2}{3}\right)^e$.

Solution 3 by Kee-Wai Lau, Hong Kong, China

For $0 < x < 1$, let $f(x)$ be the convex function $x^e - e^2\ln x$. By taking logarithms, we see that the inequality of the problem is equivalent to

$$af(c)+bf(a)+cf(b) > e. \tag{1}$$

Let $\gamma_1 = \dfrac{a}{e}, \gamma_2 = \dfrac{b}{e}$ and $\gamma_3 = \dfrac{c}{e}$. By Jensen's inequality, the left side of (1) is greater than or equal to $ef(\gamma_1 c + \gamma_2 a + \gamma_3 b) = ef\left(\dfrac{ab+bc+ca}{e}\right)$.

Since $f'(x) = \dfrac{e(x^e-e)}{x} < 0$ and
$ab+bc+ca = \dfrac{2(a+b+c)^2-(a-b)^2-(b-c)^2-(c-a)^2}{6} \le \dfrac{e^3}{3}$, so

$$f\left(\frac{ab+bc+ca}{e}\right) \ge f\left(\frac{e}{3}\right) = 1.49\cdots > 1.$$

Thus (1) holds and this completes the solution.

Solution 4 by Michel Bataille, Rouen, France

Taking logarithms and arranging, we see that the inequality is equivalent to

$$\frac{a}{e}\cdot c^e + \frac{b}{e}\cdot a^e + \frac{c}{e}\cdot b^e > 1 + e^2\left(\frac{b}{e}\cdot\ln a + \frac{c}{e}\cdot\ln b + \frac{a}{e}\cdot\ln c\right).$$

Since the functions $x \mapsto x^e$ and $x \mapsto \ln x$ are respectively convex and concave on $(0, \infty)$, Jensen's inequality yields

$$\frac{a}{e} \cdot c^e + \frac{b}{e} \cdot a^e + \frac{c}{e} \cdot b^e \geq \left(\frac{ab + bc + ca}{e}\right)^e$$

and

$$\frac{b}{e} \cdot \ln a + \frac{c}{e} \cdot \ln b + \frac{a}{e} \cdot \ln c \leq \ln\left(\frac{ab + bc + ca}{e}\right).$$

Therefore, it is sufficient to prove that

$$U^e - e^2 \ln U - 1 > 0 \tag{1}$$

where $U = \frac{ab+bc+ca}{e}$.
Since $e^2 = (a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca) \geq 3(ab + bc + ca)$, we have $U \leq \frac{e}{3}$, hence $U \in (0, 1)$.
Now, let $f(x) = x^e - e^2 \ln x - 1$. The function f satisfies $f(1) = 0$ and $f'(x) = \frac{e(x^e - e)}{x}$. It follows that f is strictly decreasing on the interval $(0, 1]$ and so $f(U) > f(1)$, which is the desired inequality (1).

Also solved by Ed Gray, Highland Beach, FL; Ioannis D. Sfikas, National and Kapodistrian University of Athens, Greece; Albert Stadler of Herrliberg, Switzerland, and the proposer.

5503: *Proposed by José Luis Díaz-Barrero, Barcelona Tech, Barcelona, Spain*

Let $a_1, a_2, \ldots, a_n$ be positive real numbers with $n \geq 2$. Prove that

$$\frac{(a_1^m a_2 + a_2^m a_3 + \cdots + a_n^m a_1)^m}{(a_1^m + a_2^m + \cdots a_n^m)^{m+1}} \leq \frac{1}{n},$$

where m is a positive integer.

Solution 1 by Michel Bataille, Rouen, France

Let $I_m(a_1, a_2, \ldots, a_n)$ denote the proposed inequality.
First we suppose that $I_1(a_1, a_2, \ldots, a_n)$ holds for all $a_1, \ldots, a_n > 0$, in other words that

$$n(a_1 a_2 + a_2 a_3 + \cdots + a_n a_1) \leq (a_1 + a_2 + \cdots + a_n)^2 \tag{1}$$

for all positive $a_1, \ldots, a_n$ and we show that if m is an integer with $m \geq 2$, then $I_m(a_1, \ldots, a_n)$ also holds for all positive $a_1, \ldots, a_n$.
Let m be an integer with $m \geq 2$ and let $a_1, \ldots, a_n > 0$. Applying (1) with $a_1^m, \ldots, a_n^m$ instead of $a_1, \ldots, a_n$, respectively, we obtain $(a_1^m + \cdots + a_n^m)^2 \geq n(a_1^m a_2^m + \cdots + a_n^m a_1^m)$ so that

$$(a_1^m + \cdots + a_n^m)^{m+1} = (a_1^m + \cdots + a_n^m)^2 (a_1^m + \cdots + a_n^m)^{m-1} \geq n(a_1^m a_2^m + \cdots + a_n^m a_1^m)(a_1^m + \cdots + a_n^m)^{m-1}.$$

But from Holder's inequality, we have

$$(a_1^m a_2^m + \cdots + a_n^m a_1^m)(a_1^m + \cdots + a_n^m)^{m-1} \geq (a_1^m a_2 + a_2^m a_3 + \cdots a_n^m a_1)^m$$

and it follows that $(a_1^m + \cdots + a_n^m)^{m+1} \geq n(a_1^m a_2 + a_2^m a_3 + \cdots a_n^m a_1)^m$, which is the desired inequality $I_m(a_1, \ldots, a_n)$.
Now, we show that (1) holds for all positive $a_1, \ldots, a_n$ if and only if $n \leq 4$.
Suppose that $n \geq 5$. If (1) holds for all $a_1, a_2, \ldots, a_n > 0$, then in particular it holds if we take $a_1 = a_2 = \cdots = a_{n-2} = \varepsilon$ and $a_{n-1} = a_n = 1$ where ε is an arbitrary positive number. This provides the inequality $n((n-3)\varepsilon^2 + 2\varepsilon + 1) \leq ((n-2)\varepsilon + 2)^2$. Letting $\varepsilon \to 0^+$, we obtain $n \leq 4$, a contradiction. Thus, we must have $n \leq 4$.
Conversely, if $n = 2$ (resp. $n = 3$, resp, $n = 4$), it is easily checked that (1) is equivalent to $(a_1 - a_2)^2 \geq 0$ (resp. $(a_1 - a_2)^2 + (a_2 - a_3)^2 + (a_3 - a_1)^2 \geq 0$, resp. $(a_1 - a_2 + a_3 - a_4)^2 \geq 0$) and so $I_1(a_1, \ldots, a_n)$ holds for all $a_1, \ldots, a_n > 0$ when $n = 2, 3$ or 4.
In conclusion, $I_m(a_1, a_2, \ldots, a_n)$ holds for any positive integer m and all positive real numbers $a_1, \ldots, a_n$ if and only if $n \leq 4$.

Solution 2 by Albert Stadler, Herrliberg, Switzerland

The statement is wrong in general. It is true for $n = 2$ and for any real $m \geq 1$, since by Hölder's inequality,

$$(x^m y + xy^m)^m = x^m y^m (x^{m-1} + y^{m-1})^m \leq x^m y^m \left((1^m + 1^m)^{\frac{1}{m}} (x^m + y^m)^{\frac{m-1}{m}} \right)^m =$$

$$= x^m y^m 2 (x^m + y^m)^{m-1} \leq \frac{1}{2}(x^m + y^m)^{m+1},$$

and the last inequality is equivalent to $x^m y^m \leq \frac{1}{4}(x^m + y^m)^2$, which is clearly true.

The statement is true as well for $n = 3$ and any real $m \geq 1$, since by Hölder's inequality,

$$(x^m y + y^m z + z^m x)^m = x^m y^m z^m \left(\frac{x^{m-1}}{z} + \frac{y^{m-1}}{x} + \frac{x^{m-1}}{z} + \frac{z^{m-1}}{y} \right)^m \leq$$

$$\leq x^m y^m z^m \left(\left(\frac{1}{z^m} + \frac{1}{x^m} + \frac{1}{y^m} \right)^{\frac{1}{m}} (x^m + y^m + z^m)^{\frac{m-1}{m}} \right)^m =$$

$$= (x^m y^m + y^m z^m + z^m x^m)(x^m + y^m + z^m)^{m-1} \leq \frac{1}{3}(x^m + y^m + z^m)^{m+1},$$

and the last inequality is equivalent to $ab + bc + ca \leq \frac{1}{3}(a + b + c)^2$, with $a = x^m, b = y^m, c = z^m$, which is clearly true, since it is equivalent to $ab + bc + ca \leq a^2 + b^2 + c^2$ (which is true because of Cauchy-Schwarz).

The problem statement is not true in general, We construct counterexamples as follows: Let $a = 1$ for $1 \leq i \leq k, a_i = 0$ for $k + 1 \leq i \leq n$. Then $(a_1^m a_2 + a_2^m a)_3 + \cdots + a_n^m a_1 = k - 1$ and $a_1^{m+1} + a_2^{m+1} + \cdots + a_n^{m+1} = k$. The stated inequality then reads as

$$(k-1)^m \leq \frac{1}{n} k^{m+1}, \tag{2}$$

which fails for an infinity of triples (k, m, n). For instance (2) is wrong for $(n - 2, 1, n)$, if $n \geq 5$, it is wrong for $(n - 3, 2, n)$, if $n \geq 8$ and it is wrong for $(n - 4, 3, n)$ if $n \geq 12$.

Purists may argue that these are not real counter-examples, since $a_i = 0$, for $k + 1 \leq i \leq n$, so that not all a_i are strictly positive. However we may replace 0 by $\epsilon > 0$ and make ϵ sufficiently small to reach the same conclusion.

Also solved by Arkady Alt; San Jose, CA; Ed Gray, Highland Beach, FL; Perfetti Paolo, Department of Mathematics, Tor Vergata University Rome, Italy; Ángel Plaza, University of Las Palmas de Gran Canaria, Spain; Ioannis D. Sfikas (two solutions), National and Kapodistrain University of Athens, Greece, and the proposer.

5504: *Proposed by Ovidiu Furdui and Alina Sîntămărian bothat the Technical University of Cluj-Napoca, Cluj-Napoca, Romania*

Let $n \geq 0$ be an integer. Calculate

$$\int_0^1 \frac{x^n}{\left\lfloor \frac{1}{x} \right\rfloor} \mathrm{d}x,$$

where $\lfloor x \rfloor$ denotes the integer part of x.

Solution 1 by Albert Stadler, Herrliberg, Switzerland

We have

$$\int_0^1 \frac{x^n}{\left\lfloor \frac{1}{x} \right\rfloor} dx = \lim_{K\to\infty} \sum_{k=1}^{K} k \int_{\frac{1}{k+1}}^{\frac{1}{k}} x^n dx = \lim_{K\to\infty} \sum_{k=1}^{K} k \int_{\frac{1}{k+1}}^{\frac{1}{k}} x^n dx = \lim_{K\to\infty} \sum_{k=1}^{K} \frac{k}{n+1} \left(\frac{1}{k^{n+1}} - \frac{1}{(k+1)^{n+1}} \right)$$

$$= \frac{1}{n+1} \lim_{K\to\infty} \sum_{k=1}^{K} \left(\frac{1}{k^n} - \frac{k+1-1}{(k+1)^{n+1}} \right) = \frac{1}{n+1} \lim_{K\to\infty} \sum_{k=1}^{K} \left(\frac{1}{k^n} - \frac{1}{(k+1)^n} + \frac{1}{(k+1)^{n+1}} \right)$$

$$= \frac{1}{n+1} \lim_{K\to\infty} \left(1 - \frac{1}{(K+1)^n} + \sum_{k=1}^{K} \frac{1}{(k+1)^{n+1}} \right) = \frac{\zeta(n+1)}{n+1}.$$

Solution 2 by Stanley Rabinowitz, Chelmsford, MA

We start by breaking the interval $(0,1)$ up into subintervals over which the function $\left\lfloor \frac{1}{x} \right\rfloor$ is constant.

$$\begin{aligned}
\int_0^1 \frac{x^n}{\left\lfloor \frac{1}{x} \right\rfloor} dx &= \sum_{k=1}^{\infty} \int_{\frac{1}{k+1}}^{\frac{1}{k}} \frac{x^n}{\left\lfloor \frac{1}{x} \right\rfloor} dx \\
&= \sum_{k=1}^{\infty} \int_{\frac{1}{k+1}}^{\frac{1}{k}} \frac{x^n}{k} dx \\
&= \sum_{k=1}^{\infty} \left[\frac{x^{n+1}}{k(n+1)} \right]_{\frac{1}{k+1}}^{\frac{1}{k}} \\
&= \frac{1}{n+1} \sum_{k=1}^{\infty} \left[\frac{\left(\frac{1}{k}\right)^{n+1}}{k} - \frac{\left(\frac{1}{k+1}\right)^{n+1}}{k} \right] \\
&= \frac{1}{n+1} \sum_{k=1}^{\infty} \left[\frac{1}{k^{n+2}} - \frac{1}{k(k+1)^{n+1}} \right]
\end{aligned}$$

$$= \frac{1}{n+1}\sum_{k=1}^{\infty}[A-B]$$

where

$$A=\sum_{k=1}^{\infty}\frac{1}{k^{n+2}}=\zeta(n+2) \quad \text{and} \quad B=\sum_{k=1}^{\infty}\frac{1}{k}\left[\frac{1}{(k+1)^{n+1}}\right]$$

and where $\zeta(n)=\displaystyle\sum_{k=1}^{\infty}\frac{1}{k^n}$ is the Riemann zeta function.

Now note that by the formula for the sum of a geometric progression,

$$\sum_{i=2}^{n+1}\frac{1}{(k+1)^i}=\frac{1}{k(k+1)}-\frac{1}{k}\left[\frac{1}{(k+1)^{n+1}}\right].$$

So

$$\begin{aligned}
B &= \sum_{k=1}^{\infty}\frac{1}{k}\left[\frac{1}{(k+1)^{n+1}}\right]\\
&= \sum_{k=1}^{\infty}\left[\frac{1}{k(k+1)}-\sum_{i=2}^{n+1}\frac{1}{(k+1)^i}\right]\\
&= \sum_{k=1}^{\infty}\left[\frac{1}{k(k+1)}\right]-\sum_{k=1}^{\infty}\left[\sum_{i=2}^{n+1}\frac{1}{(k+1)^i}\right]\\
&= \sum_{k=1}^{\infty}\left[\frac{1}{k}-\frac{1}{k+1}\right]-\sum_{i=2}^{n+1}\left[\sum_{k=1}^{\infty}\frac{1}{(k+1)^i}\right]\\
&= 1-\sum_{i=2}^{n+1}(\zeta(i)-1)\\
&= n+1-\sum_{i=2}^{n+1}\zeta(i)
\end{aligned}$$

Hence

$$\begin{aligned}
\int_0^1\frac{x^n}{\left\lfloor\frac{1}{x}\right\rfloor}\,dx &= \frac{1}{n+1}[A-B]\\
&= \frac{1}{n+1}\left[\zeta(n+2)-\left(n+1-\sum_{i=2}^{n+1}\zeta(i)\right)\right]\\
&= \frac{1}{n+1}\left[\sum_{i=2}^{n+2}\zeta(i)-(n+1)\right]\\
&= \frac{1}{n+1}\left[\sum_{i=2}^{n+2}\zeta(i)\right]-1.
\end{aligned}$$

Solution 3 by Ángel Plaza, University of Las Palmas de Gran Canaria, Spain

For $x \in (0,1]$ if $\dfrac{1}{k+1} < x \le \dfrac{1}{k}$, then $k \le \left\lfloor \dfrac{1}{x} \right\rfloor < k+1$. That is for x, $\left\lfloor \dfrac{1}{x} \right\rfloor = k$. Therefore,

$$\begin{aligned}
\int_0^1 \frac{x^n}{\lfloor \frac{1}{x} \rfloor} &= \sum_{k=1}^{\infty} \int_{1/k+1}^{1/k} \frac{x^n}{k} dx \\
&= \sum_{k=1}^{\infty} \frac{1}{k(n+1)} \left(\frac{1}{k^{n+1}} - \frac{1}{(k+1)^{n+1}} \right) \\
&= \frac{1}{n+1} \sum_{k=1}^{\infty} \left(\frac{1}{k^{n+2}} - \frac{1}{k(k+1)^{n+1}} \right) \\
&= \frac{1}{n+1} \sum_{k=1}^{\infty} \left(\frac{1}{k^{n+2}} + \frac{1}{(k+1)^{n+1}} + \cdots + \frac{1}{k+1} - \frac{1}{k} \right),
\end{aligned}$$

from where

$$\int_0^1 \frac{x^n}{\left\lfloor \dfrac{1}{x} \right\rfloor} dx = -1 + \frac{\displaystyle\sum_{j=2}^{n+2} \zeta(j)}{n+1}.$$

Solution 4 by Moti Levy, Rehovot, Israel

The first step is to substitute $y = \frac{1}{x}$ and then to split the integration range into intervals $[k, k+1]$, $\quad k \ge 1$.

$$\begin{aligned}
\int_0^1 \frac{x^n}{\lfloor \frac{1}{x} \rfloor} dx = \int_1^{\infty} \frac{y^{-n-2}}{\lfloor y \rfloor} dy &= \sum_{k=1}^{\infty} \int_k^{k+1} \frac{y^{-n-2}}{k} dy \\
&= \frac{1}{(n+1)} \sum_{k=1}^{\infty} \left(\frac{1}{k^{n+2}} - \frac{1}{k(k+1)^{n+1}} \right) \\
&= \frac{1}{(n+1)} \left(\zeta(n+2) - \sum_{k=1}^{\infty} \frac{1}{k(k+1)^{n+1}} \right).
\end{aligned}$$

Let $S_n := \sum_{k=1}^{\infty} \frac{1}{k(k+1)^n}$.

$$\begin{aligned}
S_{n+1} - S_n &= \sum_{k=1}^{\infty} \left(\frac{1}{k(k+1)^{n+1}} - \frac{1}{k(k+1)^n} \right) \\
&= \sum_{k=1}^{\infty} \frac{1}{(k+1)^n} \left(\frac{1}{k(k+1)} - \frac{1}{k} \right) \\
&= -\sum_{k=1}^{\infty} \frac{1}{(k+1)^{n+1}} = 1 - \zeta(n+1). \qquad (10)
\end{aligned}$$

It is easy to see that

$$S_1 := \sum_{k=1}^{\infty} \frac{1}{k(k+1)} = 1. \tag{11}$$

It follows from (10) and (11) that

$$S_n = n - \sum_{k=1}^{n-1} \zeta(k+1).$$

$$\begin{aligned}
\int_0^1 \frac{x^n}{\left\lfloor \frac{1}{x} \right\rfloor} dx &= \frac{1}{n+1} \left(\zeta(n+2) - S_{n+1}\right) \\
&= \frac{1}{n+1} \left(\zeta(n+2) - (n+1) + \sum_{k=1}^{n} \zeta(k+1)\right) \\
&= \frac{1}{n+1} \left(\sum_{k=2}^{n+2} \zeta(k)\right) - 1.
\end{aligned}$$

Solution 5 by David Stone and John Hawkins, Georgia Southern University, Statesboro, GA

We shall show that the value of the integral is (the average of the"first" $n+1$ values of the Riemann zeta function) minus 1; that is, $\dfrac{\zeta(2)+\zeta(3)+\ldots+\zeta(n+2)}{n+1} - 1$.

We shall need the following result, which seems interesting in its own right.
Lemma 1: For
$m \geq 1, 1 + x + x(x+1) + x(x+1)^2 + x(x+1)^3 + \ldots + x(x+1)^{m-1} = (x+1)^m$.

Proof by induction. The identity is clearly true for $m = 1$. Upon the induction hypothesis,

$$\begin{aligned}
& 1 + x + x(x+1) + x(x+1)^2 + x(x+1)^3 + \ldots + x(x+1)^{m-1} = x(x+1)^m \\
= \ & (x+1)^m + x(x+1)^m \\
= \ & (x+1)^m(1+x) \\
= \ & (x+1)^{m+1}.
\end{aligned}$$

This leads to the following result about a partial fractions decomposition.

Lemma 2:
$\dfrac{1}{k(k+1)^m} = \dfrac{1}{k} - \dfrac{1}{(k+1)^m} - \dfrac{1}{(k+1)^{m-1}} - \dfrac{1}{(k+1)^{m-2}} - \cdots - \dfrac{1}{(k+1)^2} - \dfrac{1}{k+1}$.

Proof: After clearing fractions, we see that this identity is equivalent to
$1 = (k+1)^n - k - k(k+1) - k(k+1)^2 - \ldots - k(k+1)^{n-2} - k(k+1)^{n-1}$, which is true by Lemma 1.

Now we are in position to calculate the given integral. Note that
$\left\lfloor \dfrac{1}{x} \right\rfloor = k \iff k \leq \dfrac{1}{x} < k+1 \iff \dfrac{1}{k+1} x \leq \dfrac{1}{k}$.

Thus

$$\int_{\frac{1}{k+1}}^{\frac{1}{k}} \frac{x^n}{\left\lfloor \frac{1}{x} \right\rfloor} dx = \int_{\frac{1}{k+1}}^{\frac{1}{k}} \frac{x^n}{k} dx = \frac{1}{k} \frac{1}{n+1} \left[\left(\frac{1}{k} \right)^{n+1} - \left(\frac{1}{k+1} \right)^{n+1} \right] = \frac{1}{k} \frac{1}{n+1} \left[\frac{1}{k^{n+1}} - \frac{1}{(k+1)^{n+1}} \right].$$

Therefore,

$$\int_0^1 \frac{x^n}{\left\lfloor \frac{1}{x} \right\rfloor} dx = \sum_{k=1}^{\infty} \int_{\frac{1}{k+1}}^{\frac{1}{k}} \frac{x^n}{\left\lfloor \frac{1}{x} \right\rfloor} dx = \sum_{k=1}^{\infty} \frac{1}{k} \frac{1}{n+1} \left[\frac{1}{k^{n+1}} - \frac{1}{(k+1)^{n+1}} \right]$$

$$= \frac{1}{n+1} \sum_{k=1}^{\infty} \left[\frac{1}{k^{n+2}} - \frac{1}{k(k+1)^{n+1}} \right]$$

$$= \frac{1}{n+1} \sum_{k=1}^{\infty} \left[\frac{1}{k^{n+2}} - \left\{ \frac{1}{k} - \frac{1}{(k+1)^{n+1}} - \frac{1}{(k+1)^n} - \frac{1}{(k+1)^{n-1}} - \cdots - \frac{1}{(k+1)^2} - \frac{1}{k+1} \right\} \right]$$

$$\underset{\text{by Lemma 2}}{=} \frac{1}{n+1} \sum_{k=1}^{\infty} \left[\frac{1}{k^{n+2}} - \frac{1}{k} + \frac{1}{(k+1)^{n+1}} + \frac{1}{(k+1)^n} + \frac{1}{(k+1)^{n-1}} + \cdots + \frac{1}{(k+1)^2} + \frac{1}{k+1} \right]$$

$$= \frac{1}{n} \left\{ \sum_{k=1}^{\infty} \frac{1}{k^{n+2}} + \sum_{k=1}^{\infty} \frac{1}{(k+1)^{n+1}} + \sum_{k=1}^{\infty} \frac{1}{(k+1)^n} + \cdots + \sum_{k=1}^{\infty} \frac{1}{(k+1)^2} - \sum_{k=1}^{\infty} \left(\frac{1}{k} - \frac{1}{k+1} \right) \right\}.$$

The final sum in this expression telescopes, and its sum is 1.

Each of the other sums is a shifted version of the zeta function:

$$\sum_{i=1}^{\infty} \frac{1}{(k+1)^m} = \frac{1}{2^m} + \frac{1}{3^m} + \frac{1}{4^m} + \cdots = \left(-1 + \frac{1}{1^m} \right) + \frac{1}{2^m} + \frac{1}{3^m} + \frac{1}{4^m} + \cdots = -1 = \zeta(m).$$

Therefore,

$$\int_0^1 \frac{x^n}{\left\lfloor \frac{1}{x} \right\rfloor} dx = \frac{1}{n+1} \left\{ \sum_{k=1}^{\infty} \frac{1}{k^{n+2}} + \sum_{k=1}^{\infty} \frac{1}{(k+1)^{n+1}} + \sum_{k=1}^{\infty} \frac{1}{(k+1)^n} + \cdots + \sum_{k=1}^{\infty} \frac{1}{(k+1)^2} - \sum_{k=1}^{\infty} \left(\frac{1}{k} - \frac{1}{k+1} \right) \right\}$$

$$= \frac{1}{n+1} \{ [-1 + \zeta(n+2)] + [-1 + \zeta(n+1)] + [-1 + \zeta(n)] + [-1 + \zeta(n-1)] + \cdots + [-1 + \zeta(2)] - 1 \}$$

$$= \frac{1}{n+1} \{ \zeta(2) + \zeta(3) + \cdots + \zeta(n) + \zeta(n+1) + \zeta(n+2) - n \cdot 1 - 1 \}$$

$$= \frac{1}{n+1} \{ \zeta(2) + \zeta(3) + \cdots + \zeta(n) + \zeta(n+1) + \zeta(n+2) \} - 1.$$

There are $n+1$ terms inside the braces, so we have our promised result:

$$\int_0^1 \frac{x^n}{\left\lfloor \frac{1}{x} \right\rfloor} dx =$$

(the average of the first n + 1 values of the Riemann zeta function) minus 1.

Comment: There are other variants of this answer, because the values of the zeta function for even n can be expressed in terms of the Bernoulli numbers. That would not make the answer any nicer though.

Also solved by Michel Bataille, Rouen, France; Ed Gray, Highland Beach, FL; Kee-Wai Lau, Hong Kong, China; Ioannis D. Sfikas (two solutions), National and Kapodistrian University of Athens, Greece; Perfetti Paolo, Department of Mathematics, Tor Vergata University Rome, Italy; Daniel Văcaru, Pitesti, Romania, and the proposers.

Mea Culpa

Ioannis D. Sfikas of National and Kapodistrian in University of Athens, Greece should have been credited with having solved 5495 and 5496.

Carl Libis of Columbia Southern University in Orange Beach, AL should have been credited with having solved 5497.

Correction: Problem 5514 in the November 2018 issue of this column should have been stated as:

If $a \in (0,1)$ and $b = \arcsin a$, then calculate $\lim\limits_{n\to\infty} \sqrt[n]{n!}\left(\sin\left(\dfrac{b\cdot \sqrt[n+1]{(2n+1)!!}}{\sqrt[n]{(2n-1)!!}}\right) - a\right)$.