# Problems

Ted Eisenberg, Section Editor

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This section of the Journal offers readers an opportunity to exchange interesting mathematical problems and solutions. Please send them to Ted Eisenberg, Department of Mathematics, Ben-Gurion University, Beer-Sheva, Israel or fax to: 972-86-477-648. Questions concerning proposals and/or solutions can be sent e-mail to <eisenbt@013.net>. Solutions to previously stated problems can be seen at <http://ssmj.tamu.edu>.

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*Solutions to the problems stated in this issue should be posted before February 15, 2009*

- 5038: *Proposed by Kenneth Korbin, New York, NY.*

  Given the equations

$$\begin{cases} \sqrt{1+\sqrt{1-x}} \quad -5\cdot\sqrt{1-\sqrt{1-x}} = 4\cdot\sqrt[4]{x} \ \text{ and} \\ 4\cdot\sqrt{1+\sqrt{1-y}} \quad -5\cdot\sqrt{1-\sqrt{1-y}} = \sqrt[4]{y}. \end{cases}$$

  Find the positive values of $x$ and $y$.

- 5039: *Proposed by Kenneth Korbin, New York, NY.*

  Let $d$ be equal to the product of the first $N$ prime numbers which are congruent to $1 \pmod 4$. That is

$$d = 5\cdot 13\cdot 17\cdot 29\cdots P_N.$$

  A convex polygon with integer length sides is inscribed in a circle with diameter $d$. Prove or disprove that the maximum possible number of sides of the polygon is the $N^{\text{th}}$ term of the sequence $t = (4, 8, 20, 32, 80, \cdots, t_N, \cdots)$ where $t_N = 4t_{N-2}$ for $N > 3$.

  Examples: If $N = 1$, then $d = 5$, and the maximum polygon has 4 sides $(3, 3, 4, 4)$. If $N = 2$, then $d = 5\cdot 13 = 65$ and the maximum polygon has 8 sides $(16, 16, 25, 25, 25, 25, 33, 33)$.

  *Editor's comment: In correspondence with Ken about this problem he wrote that he has been unable to prove the formula for $N > 5$; so it remains technically a conjecture.*

- 5040: *Proposed by John Nord, Spokane, WA.*

  Two circles of equal radii overlap to form a lens. Find the distance between the centers if the area in circle A that is not covered by circle B is $\dfrac{1}{3}\left(2\pi + 3\sqrt{3}\right)r^2$.

- 5041: *Proposed by Michael Brozinsky, Central Islip, NY.*

  Quadrilateral $ABCD$ (with diagonals $AC = d_1$ and $BD = d_2$ and sides $AB = s_1, BC = s_2, CD = s_3$, and $DA = s_4$) is inscribed in a circle. Show that:

$$d_1^2 + d_2^2 + d_1 d_2 > \frac{s_1^2 + s_2^2 + s_3^2 + s_4^2}{2}.$$

- 5042: *Proposed by Miquel Grau-Sánchez and José Luis Díaz-Barrero, Barcelona, Spain.*

  Let $A(z) = z^n + \sum_{k=0}^{n-1} a_k z^k$ $(a_k \neq 0)$ and $B(z) = z^{n+1} + \sum_{k=0}^{n} b_k z^k$ $(b_k \neq 0)$ be two prime polynomials with roots $z_1, z_2, \ldots, z_n$ and $w_1, w_2, \ldots, w_{n+1}$ respectively. Prove that

$$\frac{A(w_1)A(w_2)\ldots A(w_{n+1})}{B(z_1)B(z_2)\ldots B(z_n)}$$

  is an integer and determine its value.

- 5043: *Ovidiu Furdui, Toledo, OH.*

  Solve the following diophantine equation in positive integers $k$, $m$, and $n$

$$k \cdot n! \cdot m! + m! + n! = (m+n)!.$$

*Solutions*

- 5020: *Proposed by Kenneth Korbin, New York, NY.*

  Find positive numbers $x$ and $y$ such that

$$\begin{cases} x^7 - 13y = 21 \\ 13x - y^7 = 21 \end{cases}$$

**Solution 1 by Brian D. Beasley, Clinton, SC.**

Using the Fibonacci numbers $F_1 = 1$, $F_2 = 1$, and $F_n = F_{n-2} + F_{n-1}$ for each integer $n \geq 3$, we generalize the given problem by finding numbers $x$ and $y$ such that

$$\begin{cases} x^n - F_n y & = & F_{n+1} \\ F_n x + (-1)^n y^n & = & F_{n+1} \end{cases}$$

for each positive integer $n$. (The given problem is the case $n = 7$.)

We let $\alpha = (1+\sqrt{5})/2$ and $\beta = (1-\sqrt{5})/2$ and apply the Binet formula $F_n = (\alpha^n - \beta^n)/\sqrt{5}$ for each positive integer $n$ to show that we may take $x = \alpha > 0$ and $y = -\beta > 0$:

$$\alpha^n - F_n(-\beta) = \frac{\alpha^n(\beta+\sqrt{5}) - \beta^{n+1}}{\sqrt{5}} = \frac{\alpha^{n+1} - \beta^{n+1}}{\sqrt{5}} = F_{n+1};$$

$$F_n(\alpha) + (-1)^n(-\beta)^n = \frac{\alpha^{n+1} - \beta^n(\alpha - \sqrt{5})}{\sqrt{5}} = \frac{\alpha^{n+1} - \beta^{n+1}}{\sqrt{5}} = F_{n+1}.$$

**Solution 2 by David Stone and John Hawkins, Statesboro, GA.**

The solution anticipated by the poser is probably $(\alpha, -\beta)$, where $\alpha = \dfrac{1+\sqrt{5}}{2} \approx 1.618034$ is the Golden Ratio and $\beta = \dfrac{1-\sqrt{5}}{2} \approx -0.618034$ its companion (in the official terminology of *The Fibonacci Quarterly* ).

Note that:

(#) if $(x, y)$ is a solution to the system, then so is $(-y, x)$. Thus we may as well look for all solutions, not just positive solutions. We graph the system in the form

$$\begin{cases} y = \dfrac{x^7 - 21}{13} \\ y = (13x - 21)^{1/7} \end{cases}.$$

There are five points of intersection. Graphically conditional (#) appears as symmetry of intersections across the line $y = -x$ (even though the curves themselves have no such symmetry).

Although we cannot determine all solutions analytically, we have their approximate numerical values:

$$\begin{array}{rcl} (x & , & y) \\ (1.6418599 & , & 0.85866981) \\ (1.61803399 & , & 0.61803399) \\ (1.249536927 & , & -1.249536927) \\ (-0.61803399 & , & -1.61803399) \\ (-0.85866981 & , & -1.6418599) \end{array}$$

**(1)** The second and fourth solutions seem to lie on the line $y = x - 1$, suggesting that $x$ satisfies $x^7 - 13(x-1) = 21$, so $x^7 - 13x - 8 = 0$. Factoring,

$$x^7 - 13x - 8 = (x^2 - x - 1)(x^5 + x^4 + 2x^3 + 3x^2 + 5x + 8)$$

and the quadratic factor has (well-known) roots

$$\alpha = \frac{1+\sqrt{5}}{2} \approx 1.618034 \text{ and } \beta = \frac{1-\sqrt{5}}{2} \approx -0.618034.$$

Thus we actually know the second and fourth solutions are $(\alpha, -\beta)$ and $(\beta, -\alpha)$.

We verify that $(\alpha, -\beta)$ is indeed a solution to the given system. Note that by the first well-known relationship to the Fibonacci numbers, $\alpha^n = \alpha F_n + F_{n-1}$, we have $\alpha^7 = \alpha F_7 + F_6 = 13\alpha + 8$.

Now, substituting into the first equation:

$$\begin{aligned} \alpha^7 - 13(-\beta) &= \alpha^7 - 13(\alpha - 1) \\ &= \alpha^7 - 13\alpha + 13 \\ &= 13\alpha + 8 - 13\alpha + 13 = 21, \text{as desired.} \end{aligned}$$

It is also straight forward to verify the second equation: $13\alpha - (-\beta)^7 = 21$, using $\alpha\beta = -1$.

**(2)** The third solution lies on the line $y = -x$, so $x$ is the sole real zero of $x^7 + 13x - 21 = 0$. This polynomial equation is not solvable in radicals – according to Maple, the Galois group of $x^7 + 13x - 21$ is $S_7$, which is not a solvable group. Hence, an approximation is probably the best we can do (barring some ingenious treatment employing transcendental functions.)

Unfortunately, we do not have any analytic characterization of the first and fifth solutions.

A final comment: the problem involves the exponent 7 and the Fibonacci numbers $F_7 = 13$ and $F_8 = 21$, so there is almost certainly a more general version with solution $(\alpha, -\beta)$.

**Also solved by Bruno Salgueiro Fanego, Viveiro, Spain; Paul M. Harms, North Newton, KS; Peter E. Liley, Lafayette, IN; Charles McCracken, Dayton, OH; Boris Rays, Chesapeake, VA; David C. Wilson, Winston-Salem, NC, and the proposer.**

- 5021: *Proposed by Kenneth Korbin, New York, NY.*

  Given
$$\frac{x+x^2}{1-34x+x^2} = x + 35x^2 + \cdots + a_n x^n + \cdots$$

  Find an explicit formula for $a_n$.

  **Solution by David E. Manes, Oneonta, NY.**

  An explicit formula for $a_n$ is given by
$$a_n = -\frac{1}{8}\Big[(4-3\sqrt{2})(17+12\sqrt{2})^n + (4+3\sqrt{2})(17-12\sqrt{2})^n\Big].$$

  Let $F(x) = \dfrac{x+x^2}{1-34x+x^2}$ be the generating function for the sequence $\Big(a_n\Big)_{n\geq 1}$, where $a_1 = 1$ and $a_2 = 35$.
  Then the characteristic equation is $\lambda^2 - 34\lambda + 1 = 0$, with roots $r_1 = 17 + 12\sqrt{2}$ and $r_2 = 17 - 12\sqrt{2}$.

  Therefore,
$$a_n = \alpha\Big(17+12\sqrt{2}\Big)^n + \beta\Big(17-12\sqrt{2}\Big)^n$$

  for some real numbers $\alpha$ and $\beta$. From the initial conditions one obtains
$$\begin{aligned} 1 &= \alpha\Big(17+12\sqrt{2}\Big) + \beta\Big(17-12\sqrt{2}\Big) \\ 35 &= \alpha\Big(17+12\sqrt{2}\Big)^2 + \beta\Big(17-12\sqrt{2}\Big)^2. \end{aligned}$$

  The solution for this system of equations is
$$\begin{aligned} \alpha &= -\frac{1}{8}\Big(4-3\sqrt{2}\Big) \\ \beta &= -\frac{1}{8}\Big(4+3\sqrt{2}\Big). \end{aligned}$$

  Hence, if $n \geq 1$, then
$$a_n = -\frac{1}{8}\Big[(4-3\sqrt{2})(17+12\sqrt{2})^n + (4+3\sqrt{2})(17-12\sqrt{2})^n\Big].$$

  **Also solved by Brian D. Beasley, Clinton, SC; Dionne T. Bailey, Elsie M. Campbell, and Charles Diminnie (jointly), San Angelo, TX; Bruno Salgueiro**

**Fanego, Viveiro, Spain; Paul M. Harms, North Newton, KS; Kee-Wai Lau, Hong Kong, China; Boris Rays, Chesapeake, VA; David Stone and John Hawkins, Statesboro, GA; David C. Wilson, Winston-Salem, NC, and the proposer.**

- 5022: *Proposed by Michael Brozinsky, Central Islip, NY.*

Show that

$$\sin\left(\frac{x}{3}\right)\sin\left(\frac{\pi+x}{3}\right)\sin\left(\frac{2\pi+x}{3}\right)$$

is proportional to $\sin(x)$.

**Solution 1 by José Hernández Santiago, (student, UTM), Oaxaca, México.**

From the well-known identity $\sin 3\theta = 3\cos^2\theta\sin\theta - \sin^3\theta$, we derive that

$$\begin{aligned}
\sin 3\theta &= 4\sin\theta\Big(\frac{3}{4}\cos^2\theta - \frac{1}{4}\sin^2\theta\Big)\\
&= 4\sin\theta\Big(\frac{\sqrt{3}}{2}\cos\theta - \frac{1}{2}\sin\theta\Big)\Big(\frac{\sqrt{3}}{2}\cos\theta + \frac{1}{2}\sin\theta\Big)\\
&= 4\sin\theta\sin\left(\frac{\pi}{3}-\theta\right)\sin\left(\frac{\pi}{3}+\theta\right).
\end{aligned}$$

When we let $\theta = \dfrac{x}{3}$, the latter formula becomes:

$$\sin 3\Big(\frac{x}{3}\Big) = 4\sin\left(\frac{x}{3}\right)\sin\left(\frac{\pi-x}{3}\right)\sin\left(\frac{\pi+x}{3}\right) \qquad (1)$$

Now, the fact that

$$\begin{aligned}
\sin\left(\frac{x+2\pi}{3}\right) &= \sin\left(\frac{x-\pi}{3}+\pi\right)\\
&= \sin\left(\frac{x-\pi}{3}\right)\cos\pi\\
&= \sin\left(\frac{\pi-x}{3}\right)
\end{aligned}$$

allows us to put (1) in the form

$$\sin x = 4\sin\left(\frac{x}{3}\right)\sin\left(\frac{\pi+x}{3}\right)\sin\left(\frac{x+2\pi}{3}\right);$$

and clearly this is equivalent to what the problem asked us to demonstrate.

**Solution 2 by Kee-Wai Lau, Hong Kong, China.**

Since

$$\begin{aligned}
\sin\left(\frac{\pi+x}{3}\right)\sin\left(\frac{2\pi+x}{3}\right) &= \frac{1}{2}\Big(\cos\left(\frac{\pi}{3}\right)+\cos\left(\frac{2x}{3}\right)\Big)\\
&= \frac{1}{2}\Big(\frac{1}{2}+1-2\sin^2\left(\frac{x}{3}\right)\Big)\\
&= \frac{1}{4}\Big(3-4\sin^2\left(\frac{x}{3}\right)\Big),
\end{aligned}$$

so

$$\sin\left(\frac{x}{3}\right)\sin\left(\frac{\pi+x}{3}\right)\sin\left(\frac{2\pi+x}{3}\right)=\frac{1}{4}\left(3\sin\left(\frac{x}{3}\right)-4\sin^3\left(\frac{x}{3}\right)\right)=\frac{1}{4}\sin(x),$$

which is proportional to $\sin(x)$.

**Also solved by Brian D. Beasley, Clinton, SC; John Boncek, Montgomery, AL; Elsie M. Campbell, Dionne T. Bailey, and Charles Diminnie (jointly), San Angelo, TX; Michael C. Faleski, University Center, MI; Bruno Salgueiro Fanego, Viveiro, Spain; Paul M. Harms, North Newton, KS; Jahangeer Kholdi, Portsmouth, VA; Kenneth Korbin, NY, NY: Peter E. Liley, Lafayette, IN; David E. Manes, Oneonta, NY; Charles, McCracken, Dayton, OH; John Nord, Spokane, WA; Paolo Perfetti, Mathematics Department, University "Tor Vergata", Rome, Italy; Boris Rays, Chesapeake, VA; David Stone and John Hawkins (jointly), Statesboro, GA; David C. Wilson, Winston-Salem, NC, and the proposer.**

- 5023: *Proposed by M.N. Deshpande, Nagpur, India.*

Let $A_1A_2A_3\cdots A_n$ be a regular $n-$gon ($n\geq 4$) whose sides are of unit length. From $A_k$ draw $L_k$ parallel to $\text{A}_{k+1}A_{k+2}$ and let $L_k$ meet $L_{k+1}$ at $T_k$. Then we have a "necklace" of congruent isosceles triangles bordering $A_1A_2A_3\cdots A_n$ on the inside boundary. Find the total area of this necklace of triangles.

**Solution 1 by Paul M. Harms, North Newton, KS.**

In order that the "necklace" of triangles have the $n-$gon as an inside boundary, it appears that line $L_k$ (through $A_k$) should be parallel to $A_{k-1}A_{k+1}$ rather than $A_{k+1}A_{k+2}$. With this interpretation in mind, we now consider the $n$ isosceles triangles with a vertex at the center of the $n-$gon and the opposite side being a side of unit length. The measure of the central angles are $360^o/n$. The angle inside the n-gon at the intersection of 2 unit sides is twice one of the equal angles of the isosceles triangles with a vertex at the center of the $n-$gon, so it has a degree measure of $180^\text{o}-(360^\text{o}/n)$.

The isosceles triangle $A_{k-1}A_kA_{k+1}$ has two equal angles (opposite the sides of unit length) with a measure of

$$\frac{1}{2}\Big(180^o-(180^o-(360^o/n))\Big)=\frac{180^o}{n}.$$

A side of length one intersects the two parallel lines ($A_{k-1}A_{k+1}$) and the line parallel to it through $A_k$. Using equal angles for a line intersecting parallel lines, we see that the equal angles in one necklace isosceles triangle has a measure of $180^o/n$.

Using the side of length one as a base, the area of one necklace triangle is

$$\frac{1}{2}(\text{base})\cdot(\text{height})=\frac{1}{2}(1)\left(\frac{1}{2}\tan(180^\text{o}/n)\right)=\frac{1}{4}\tan(180^\text{o}/n).$$

The total area of $n$ necklace triangles is $\dfrac{n}{4}\tan(180^o/n)$. It is interesting to note that the total area approaches $\pi/4$ as $n$ gets large.

**Solution 2 by David Stone and John Hawkins, Statesboro, GA.**

*David and John looked at the problem a bit differently than the other solvers. They wrote:* "In order to get a clearer picture of what is going on, we introduce additional points that we will call $B_k$, where we define $B_k$ to be the intersection of $L_k$ and $L_{k-2}$, for $3 \le k \le n$ and the intersection of $L_k$ and $L_{k+n-2}$ for $k = 1$ or 2."

*Doing this gave them a* "necklace of isosceles triangles with bases along the interior boundary of the polygon: $\triangle A_1B_1A_2, \triangle A_2B_2A_3, \triangle A_3B_3A_4, \cdots, \triangle A_nB_nA_1$." *(Note that by doing this $A_kT_k$ does pass through $A_{k+3}$.)*

*They went on:* "It is not clear that this was the intended necklace, because these triangles do not involve the points $T_k$. Let's call this the Perimeter Necklace."

There is a second necklace of isosceles triangle whose bases do involve the points $T_k:\triangle T_1B_3T_2,\ \triangle T_2B_4T_3,\ \triangle T_3B_5T_4,\ \cdots,\ \triangle T_{n-2}B_nTn_1,\ \triangle T_{n-1}B_1T_1,\ \triangle T_nB_2T_1$. Let's call this the Inner Necklace.

*They then found the areas for both necklaces and summarized their results as follows:*

$n = 4$: Area of Perimeter Necklace $= 0$. No Inner Necklace.

$n = 5$:

$$\begin{aligned}\text{Area of Perimeter Necklace} &= \frac{5}{4}\tan\left(\frac{\pi}{5}\right)\\ \text{Area of Inner Necklace} &= \frac{5}{4}\left(1-\tan^2\frac{\pi}{5}\right)\sin\left(\frac{\pi}{5}\right)\end{aligned}$$

$n = 6$

$$\begin{aligned}\text{Area of Perimeter Necklace} &= \frac{3}{2}\tan\left(\frac{\pi}{3}\right) = \frac{3\sqrt{3}}{2}\\ \text{Area of Inner Necklace} &= 0.\end{aligned}$$

$n > 5$

$$\begin{aligned}\text{Area of Perimeter Necklace} &= \frac{n}{4}\tan\left(\frac{2\pi}{n}\right)\\ \text{Area of Inner Necklace} &= \frac{\pi}{4}\left(4\sin\frac{2\pi}{n}\cos\frac{2\pi}{n}-4\sin\frac{2\pi}{n}+\tan\frac{2\pi}{n}\right)\end{aligned}$$

Note that these give the correct results for n=6.

*Then they used Excel to compute the areas of the necklaces for various values of n, and proved that for large values of n, the ratio of the areas approaches one.*

| $n$ | $PerimeterNecklace$ | $InnerNecklace$ |
|---|---|---|
| 6 | 2.59876211 | 0 |
| 10 | 1.81635632 | 0.693786379 |
| 100 | 1.572747657 | 1.561067973 |
| 500 | 1.570879015 | 1.570382935 |

$$\lim_{n\to\infty} \frac{\frac{n}{4}\left(4\sin\frac{2\pi}{n}\sin\frac{2\pi}{n}-4\sin\frac{2\pi}{n}+\tan\frac{2\pi}{n}\right)}{\frac{n}{4}\tan\frac{2\pi}{n}} = 1.$$

**Also solved by Bruno Salgueiro Fanego, Viveiro, Spain; Michael N. Fried, Kibbutz Revivim, Israel; Grant Evans (student, Saint George's School), Spokane, WA; Boris Rays, Chesapeake, VA, and the proposer.**

- 5024: *Proposed by Luis Díaz-Barrero and Josep Rubió-Masseguú, Barcelona, Spain.*

Find all real solutions to the equation

$$\sqrt{1+\sqrt{1-x}}-2\sqrt{1-\sqrt{1-x}}=\sqrt[4]{x}.$$

**Solution by Jahangeer Kholdi, Portsmouth, VA.**

Square both sides of the equation, simplify, and then factor to obtain

$$5(1-\sqrt{x})=3\sqrt{1-x}.$$

Squaring again gives $17x-25\sqrt{x}+8=0$, and now using the quadratic formula gives $x=1$ and $x=\dfrac{64}{289}$. But $x=1$ does not satisfy the original equation. The only real solution to the original equation is $x=\dfrac{64}{289}$.

**Also solved by Brian D. Beasley, Clinton, SC; John Boncek, Montgomery, AL; Elsie M. Campbell, Dionne T. Bailey, and Charles Diminnie (jointly), San Angelo, TX; Matt DeLong, Upland, IN; Grant Evans (student, Saint George's School), Spokane, WA; Michael C. Faleski, University Center, MI; Bruno Salgueiro Fanego, Viveiro, Spain; Paul M. Harms, North Newton, KS; Kenneth Korbin, NY, NY; Kee-Wai Lau, Hong Kong, China; Peter E. Liley, Lafayette, IN; David E. Manes, Oneonta, NY; Charles McCracken, Dayton, OH; Wattana Namkaew (student, Nakhon Ratchasima Rajabhat University), Thailand; John Nord, Spokane, WA; Paolo Perfetti, Mathematics Department, University "Tor Vergata", Rome, Italy; Boris Rays, Chesapeake, VA; David Stone and John Hawkins (jointly), Statesboro, GA; David C. Wilson, Winston-Salem, NC, and the proposers.**

- 5025: *Ovidiu Furdui, Toledo, OH.*

Calculate the double integral

$$\int_0^1\int_0^1\{x-y\}dxdy,$$

where $\{a\}=a-[a]$ denotes the fractional part of $a$.

**Solution by R. P. Sealy, Sackville, New Brunswick, Canada.**

$$\begin{aligned}\int_0^1\int_0^1\{x-y\}dxdy &= \int_0^1\int_0^x\{x-y\}dydx+\int_0^1\int_0^y\{x-y\}dxdy\\ &= \int_0^1\int_0^x(x-y)dydx+\int_0^1\int_0^y(x-y+1)dxdy\\ &= \int_0^1\left(xy-\frac{y^2}{2}\right)\Bigg|_0^x dx+\int_0^1\left(\frac{x^2}{2}-xy+x\right)\Bigg|_0^y dy\\ &= \int_0^1\frac{x^2}{2}dx+\int_0^1\left(y-\frac{y^2}{2}\right)dy\end{aligned}$$

$$
\begin{aligned}
&= \left.\frac{x^3}{6}\right|_0^1 + \left.\left(\frac{y^2}{2} - \frac{y^3}{6}\right)\right|_0^1 \\
&= \frac{1}{2}.
\end{aligned}
$$

**Also solved by Elsie M. Campbell, Dionne T. Bailey, and Charles Diminnie (jointly), San Angelo, TX; Matt DeLong, Upland, IN; Michael C. Faleski, University Center, MI; Bruno Salgueiro Fanego, Viveiro, Spain; Paul M. Harms, North Newton, KS; Nate Kirsch and Isaac Bryan (jointly, students at Taylor University), Upland, IN; Kee-Wai Lau, Hong Kong, China; Matthew Hussey, Rachel DeMeo, Brian Tencher (jointly, students at Taylor University), Upland, IN; Paolo Perfetti, Mathematics Department, University "Tor Vergata", Rome, Italy; Nicki Reishus, Laura Schindler, Landon Anspach and Jessi Byl (jointly, students at Taylor University), Upland, IN; José Hernández Santiago (student, UTM), Oaxaca, México, Boris Rays, Chesapeake, VA; David Stone and John Hawkins (jointly), Statesboro, GA, and the proposer.**